A critical comparison of formulations for the computation of induced ELF fields into the in human body

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***Abstract*—The Finite Element method is used for computing induced ELF fields into the human body. Many formulations have been proposed, basing upon different approximations. The purpose of this paper is to review critically the different approximations. The performed simulations show that as long as only ELF fields are involved, the classical approximations hold.**

I. Introduction

The finite element method (FEM) is used for evaluating the induced currents into the human body by extremely low frequencies (ELF) electric and magnetic fields. Many formulations, developed under different assumptions, are available. The most common approximations which are done are: i) the reaction field due to the presence of eddy currents into the human body are neglected, ii) displacement currents inside the human body are negligible compared with conduction currents, iii) both exposure cases are considered separately (electric and magnetic fields are decoupled) [1]. It is observed that, with the same computational phantoms, these formulations produce somehow different results [2]. The purpose of this work is to examine in details the validity of these commonly used approximations. To this aim, we have considered a general formulation, where none of these approximation is used, so that all the other studied formulations can be deduced from this one by negliging some terms.

II. Formulations

A. Complete formulation

We start from a general $\overrightarrow{A} - \phi$ formulation, where displacement currents are taken into account. All the $\overrightarrow{E}$-conform formulations can be deduced by this one, namely the classical $\overrightarrow{A} - \phi$ formulation [3], the $\phi - \overrightarrow{A}$ formulation [1], and the dielectric formulation.

The $\overrightarrow{A} - \phi$ formulation in the frequency domain and without any approximation inside a bounded domain $\Omega \subset \mathbb{R}^3$ reads:

$$\begin{cases} \overrightarrow{\mathrm{curl}}\,(\nu\,\overrightarrow{\mathrm{curl}}\,\overrightarrow{A}) - (\sigma + i\omega\epsilon)(-i\omega\overrightarrow{A} - \overrightarrow{\mathrm{grad}}\,\phi) = 0 \\ \mathrm{div}\left[(\frac{\sigma}{i\omega} + \epsilon)(-i\omega\overrightarrow{A} - \overrightarrow{\mathrm{grad}}\,\phi)\right] = 0 \end{cases} \quad (1)$$

where $\overrightarrow{B} = \overrightarrow{\mathrm{curl}}\,\overrightarrow{A}$, $\overrightarrow{E} = -i\omega\overrightarrow{A} - \overrightarrow{\mathrm{grad}}\,\phi$.

Equation 1 can be written in a weak form as:

$$\begin{cases} \int_\Omega(\nu\,\overrightarrow{\mathrm{curl}}\,\overrightarrow{A}).\,\overrightarrow{\mathrm{curl}}\,\overrightarrow{A}' d\Omega + \\ \int_\Omega(\sigma + i\omega\epsilon)(i\omega\overrightarrow{A} + \overrightarrow{\mathrm{grad}}\,\phi).\overrightarrow{A}' d\Omega = \\ \int_\Gamma(\nu\,\overrightarrow{\mathrm{curl}}\,\overrightarrow{A}) \wedge \overrightarrow{n}.\overrightarrow{A}' d\Gamma \\ \\ \int_\Omega(\frac{\sigma}{i\omega} + \epsilon)(i\omega\overrightarrow{A} + \overrightarrow{\mathrm{grad}}\,\phi).\,\overrightarrow{\mathrm{grad}}\,\phi' d\Omega = \\ \int_\Gamma(\frac{\sigma}{i\omega} + \epsilon)(i\omega\overrightarrow{A} + \overrightarrow{\mathrm{grad}}\,\phi).\,\overrightarrow{n}.\phi' d\Gamma \end{cases} \quad (2)$$

where $\Gamma = \partial\Omega$ and $\overrightarrow{n}$ the exterior unit normal to Γ. Boundary conditions have to be adapted to the studied exposure in order to take into account a source term:

- for a magnetic exposure:

$$(\overrightarrow{A} \wedge \overrightarrow{n})_{|\Gamma} \text{ fixed} \ , \ (-i\omega\overrightarrow{A} - \overrightarrow{\mathrm{grad}}\,\phi)_{|\Gamma}.\overrightarrow{n} = 0 \quad (3)$$

- for an electric exposure:

$$(\overrightarrow{A} \wedge \overrightarrow{n})_{|\Gamma} = 0 \ , \ \phi_{|\partial\Omega} \text{ fixed on } \Gamma_d \subset \Gamma \quad (4)$$

B. Magnetic formulations

For magnetic exposure, two main approximations are considered: displacement currents and reaction field from the human body are neglected. The classical $\overrightarrow{A} - \phi$ formulation only neglects displacement currents ($\omega\epsilon \ll 1$):

$$\begin{cases} \int_\Omega(\nu\,\overrightarrow{\mathrm{curl}}\,\overrightarrow{A}).\,\overrightarrow{\mathrm{curl}}\,\overrightarrow{A}' d\Omega + \\ \int_\Omega \sigma(i\omega\overrightarrow{A} + \overrightarrow{\mathrm{grad}}\,\phi).\overrightarrow{A}' d\Omega = 0 \\ \\ \int_\Omega(\frac{\sigma}{i\omega} + \epsilon)(i\omega\overrightarrow{A} + \overrightarrow{\mathrm{grad}}\,\phi).\,\overrightarrow{\mathrm{grad}}\,\phi' d\Omega = 0 \\ \\ (\overrightarrow{A} \wedge \overrightarrow{n})_{|\Gamma} \text{ fixed} \ , \ (-i\omega\overrightarrow{A} - \overrightarrow{\mathrm{grad}}\,\phi)_{|\Gamma}.\overrightarrow{n} = 0 \end{cases} \quad (5)$$

The formulation $\phi - \overrightarrow{A}$ neglects displacement currents and reaction field. Therefore computational domain Ω can be limited the human body, and $\overrightarrow{A} = \overrightarrow{A_s}$ (which is assumed to be known) is used as source term in the second equation:

$$\int_\Omega \frac{\sigma}{i\omega}(i\omega\overrightarrow{A_s} + \overrightarrow{\mathrm{grad}}\,\phi).\,\overrightarrow{\mathrm{grad}}\,\phi' d\Omega = 0 \quad (6)$$

C. Electric formulation

The "dielectric" formulation [1] only considers the electric field and totally disregards the magnetic field by imposing $\overrightarrow{A} = 0$, so that ϕ must verify:

$$\begin{cases} \int_\Omega(\frac{\sigma}{i\omega} + \epsilon)\,\overrightarrow{\mathrm{grad}}\,\phi.\,\overrightarrow{\mathrm{grad}}\,\phi' d\Omega = 0 \\ \phi_{|\partial\Omega} \ \text{ fixed on } \Gamma_d \\ (\frac{\sigma}{i\omega} + \epsilon)\,\overrightarrow{\mathrm{grad}}\,\phi_{|\partial\Omega}.\overrightarrow{n} = 0 \ \text{ on } \Gamma \backslash \Gamma_d \end{cases} \quad (7)$$

Starting from the same "complete" formulation discretized by using mixed finite elements, comparisons with dedicated formulations are possible in order to assess the impact of approximations on the solution.

III. Magnetic exposure

For each exposure case, the results are obtained with the ANSOFT human body model (Fig. 1), which is composed of 29 different tissues with different conductivities [4] and is discretized by using $NE = 7 \cdot 10^5$ tetrahedra. The in-

Fig. 1. Human body model - Distribution of induced currents (A/m^2)

duced currents are calculated with the "complete" (2) and the classical $\overrightarrow{A} - \phi$ formulation (5). In the latter case, a much faster convergency is observed. In order to compare these two formulations, we define the normalized discrepancy: $\Delta J = \frac{||\overrightarrow{J}_1|| - ||\overrightarrow{J}_2||}{\max||\overrightarrow{J}||} \times 100\%$, where $\overrightarrow{J}_1$ et $\overrightarrow{J}_2$ are computed with these two formulations, and $\max||\overrightarrow{J}||$ is the maximum current density computed with (2). For each tetrahedron, the quality q defined as in [2] and the discrepancy in the barycenter are computed, so we obtain NE couples $\{\Delta J_e, q_e\}_{e=1...NE}$. These couples are ordered in ascending order with respect of q_e. Tetrahedra are grouped in classes, the quality of which spans within $[10k\%, 10(k+1)\%]$ (with $k = 0 \dots 9$) and the average value of the discrepancy $\langle \Delta J \rangle$ is computed for each class. In Fig. 2 the average ΔJ is plotted for each class together with the quality q (the continuous line). One observes that $\langle \Delta J \rangle$ is correlated with the quality of the elements (the bigger is the quality q, the smaller is the average discrepancy $\langle \Delta J \rangle$). However, within a same class of elements very large variations of ΔJ are observed. The same approach has been used for comparing the classical $\overrightarrow{A} - \phi$ formulation (5) with the $\phi - \overrightarrow{A}$ formulation (6). Both formulations neglige displacement currents, but $\phi - \overrightarrow{A}$ formulation negliges also the reaction field due to the human body. The average discrepancy is depicted in Fig. 3. In both cases the computed discrepancies are quite negligible: this confirms the validity of the approximations employed in (5) and (6).

IV. Conclusion

The calculation of induced currents in human body is traditionally performed by using different "dedicated" formulations, which introduce some approximations with respect of

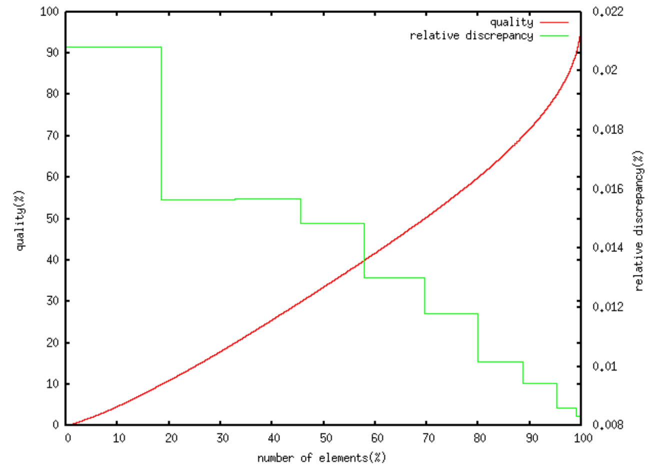

Fig. 2. Average discrepancy $\langle \Delta J \rangle$ between the complete and classical $\overrightarrow{A} - \phi$ formulations, and quality of the elements.

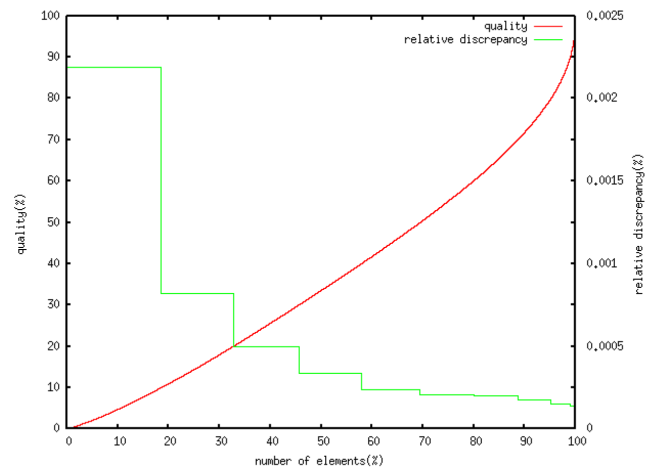

Fig. 3. Average discrepancy $\langle \Delta J \rangle$ between the classical and dedicated $\overrightarrow{A} - \phi$ formulations, and quality of the elements.

the original problem. These numerical experiments show that the error introduced by these approximations is negligible. Furthermore, high discrepancies between formulations are correlated with low quality elements (even if this correlation is rather weak), in that it holds only for the average discrepancy, but not for each element. In the full paper the case of exposure to electric field will also be adressed.

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